

Universal Level Dynamics of Complex Systems

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We study the evolution of the distribution of eigenvalues of a $N \times N$ matrix subject to a random perturbation drawn from (i) a generalized Gaussian ensemble (ii) a non-Gaussian ensemble with a measure variable under the change of basis. It turns out that, in the case (i), a redefinition of the parameter governing the evolution leads to a Fokker-Planck equation similar to the one obtained when the perturbation is taken from a standard Gaussian ensemble (with invariant measure). This equivalence can therefore help us to obtain the correlations for various physically-significant cases modeled by generalized Gaussian ensembles by using the already known correlations for standard Gaussian ensembles.

For large N -values, our results for both cases (i) and (ii) are similar to those obtained for Wigner-Dyson gas as well as for the perturbation taken from a standard Gaussian ensemble. This seems to suggest the independence of evolution, in thermodynamic limit, from the nature of perturbation involved as well as the initial conditions and therefore universality of dynamics of the eigenvalues of complex systems.

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I. INTRODUCTION

Many important physical properties of complex quantum systems can be studied by analyzing the statistical response of eigenvalues to an external perturbation. It turns out that the evolution of the eigenvalues of the system as a function of the strength of the perturbation is universal but only after an appropriate normalization of the perturbing potential [1,2]. Various approaches have been adopted for a better understanding of this phenomena e.g. non-linear σ model [3], random matrix theory (RMT) [4], semi-classical techniques [5] etc. Among these, a special class of Gaussian ensembles of RMT have been used very successfully in modelling the short energy-range behaviour of eigenvalues [4]. The probability density of a matrix in this ensemble depends only on the functions invariant under change of basis e.g trace of the matrix thus making it easier to calculate the eigenvalue distribution. However it has been conjectured that the local statistical properties of a few eigenvalues of a random hermitian matrix are independent of the details of the distribution of the matrix elements apart from a few broad characterizations such as real symmetric versus complex hermitian etc [4]. Therefore the success of RMT in modelling the energy-level behaviour of complex systems should not be restricted only to the standard Gaussian ensembles.

In this analytical work, we study this conjecture by examining the dynamics of eigenvalues of a quantum hamiltonian under a random perturbation belonging to a class of (i) a general Gaussian ensembles and (ii) a non-Gaussian ensemble with logarithm of the distribution given by a polynomial. In both cases, the probability density of a matrix is chosen to contain the basis-dependent functions and therefore is no longer invariant under the basis-transformation. By integrating over all perturbations from such an ensemble, we show that, for large N , the probability density of eigenvalues evolves according to a Fokker-Planck (F-P) equation similar to the one proposed by Dyson for Wigner-Dyson gas [6]. The strength of the perturbation acts here as a time like coordinate. For finite N , this analogy seems to survive only when the dynamics is considered as a function of the perturbation strength as well as the variances of the matrix elements of the perturbation.

The statistical response of eigenvalues to a changing perturbation was studied by Simons and coworkers [1] by using a different approach, namely, non-linear σ model. By considering a hamiltonian $H = H_0 + xV$ with V as the perturbation and x its strength, they explicitly calculate the autocorrelation function of the energy eigenvalues for disordered systems within the so called Gaussian approximation (or zero mode approximation). Other results pertain to the numerical study of perturbed quantum chaotic systems, such as the chaotic billiard with a varying magnetic flux [7]. As shown in their work, the universality manifests itself among the density correlators only after the normalization of eigenvalues λ_i by mean level spacing and that of perturbation by rms "velocity" of eigenvalues $\langle (\partial \lambda_i / \partial x)^2 \rangle$; by universality one means here the independence of the bulk correlators in a many body system from boundary effects. They also conjectured the asymptotic correspondence of the two-variable parametric density correlators of complex quantum systems with the corresponding time-dependent particle-density correlators for the Sutherland hamiltonian [8]. The connection between the static correlators (for a given parameter value) and the ground state correlators of Sutherland is already well known [1].

This correspondence was analytically shown in a recent work by Narayan and Shastri [9]. They studied the evolution of the distribution of eigenvalues of a $N \times N$ matrix H subject to a random perturbing matrix V taken from a standard Gaussian (SG) ensemble and showed that it leads to the F-P equation postulated by Dyson. Within this model, they proved the equivalence between the space-time correlators of the ground state dynamics of an integrable 1-D interacting many body quantum system (the Calogero-Sutherland system) and the second order parametric correlations of H . This analogy follows because the Dyson's F-P equation is equivalent, under a Wick rotation, to the quantum mechanics of Calogero model which, in thermodynamic limit, has identical bulk properties as the Sutherland system [9,10].

Our present work extends this analogy further to a more general model of hamiltonians H . We proceed as follows. For the clarification of our ideas, we first study, in section II, the distribution of eigenvalues $P(\mu, \tau)$ for an arbitrary initial condition H_0 and the perturbation V taken from a generalized Gaussian ensemble (later referred as GG case). We obtain a partial differential equation governing its evolution which, after certain parametric redefinitions, turns out to be formally the same as the Fokker-Planck equation governing the evolution of Wigner-Dyson (WD) gas. As the solution of F-P equation as well as various correlations in the latter case are already known for many initial conditions, this helps us to obtain $P(\mu, \tau)$ as well as the correlations in the GG case, as given in section III. The section IV deals with the evolution of eigenvalues under a more general perturbation. We are motivated to study this case because the claim about universality of the dynamics can only be made if the equivalence is proved, at least in thermodynamic limit, for a V with distribution of more general nature. We conclude in section V which is followed by an appendix containing the proofs of some of the results used in this paper.

II. GENERALIZED GAUSSIAN CASE

Our interest is in the evolution of eigenvalues of H under a random perturbation V with initial state H_0 as in Ref.[9]. Here both H_0 and V are hermitian and can be assumed to have same mean spacing without any loss of generality. To ensure the same mean spacing for H also, we need to modify the parameterization of our perturbation strength x [9,10,11] from $H(x) = H_0 + xV$ to $H(x) = H_0 \text{Cos}(\Omega x) + V\Omega^{-1} \text{Sin}(\Omega x)$ with $\Omega \propto \frac{1}{\sqrt{N}}$. Since $\Omega \rightarrow 0$ for large N , the reparameterization of the perturbation strength is inconsequential for finite x . The distribution $\rho(V)$ of matrix V is still chosen to be a Gaussian, $\rho(V) = C \exp(-\sum_{k \leq l} \alpha_{kl} V_{kl}^2)$ with C as the normalization constant. So far we are in the same situation as in Ref.[9] but now we depart and allow the variances for diagonal and off-diagonal matrix elements to be arbitrary (later referred as GG case) unlike the SG case where variance of the diagonal ones was twice of the off-diagonal ones.

As indicated in [9], the evolution of eigenvalues of H as a function of x for any perturbation V can be expressed as a set of first order differential equations. They turn out to be analogous to the equations of motion of a classically integrable system with $2N + \beta N(N-1)/2$ degrees of freedom (with β a symmetry dependent parameter). In principle, the distribution of eigenvalues can be obtained from these equations and its evolution can be studied but the coupling between the equations makes such a method very difficult. Thus we adopt another approach, the one used in [10] for studying the symmetric Gaussian case. As shown in [9,10], the proper form of F-P equation can be achieved only after a further reparametrization, namely, $\tau = -\Omega^{-2} \ln \text{Cos}(\Omega x)$ and by studying the evolution in terms of τ . This suggests us to use the same parametrization for our case also which gives

$$H(\tau) = H_0 f h + V h \Omega^{-1} \quad (1)$$

with $h = \sqrt{1 - e^{-2\Omega^2 \tau}}$ and $f h = e^{-\Omega^2 \tau}$.

Given an ensemble of the matrices V distributed with a probability density $\rho(V)$ in the Hermitian matrix space, let $P(\mu, \tau)$ be the probability of finding eigenvalues $\lambda_i[V]$ of H between μ_i and $\mu_i + d\mu_i$ at "time" τ for an arbitrarily chosen H_0 which can be expressed as follows,

$$P(\mu_i, \tau) = \int \prod_{i=1}^N \delta(\mu_i - \lambda_i[V]) \rho(V) dV \quad (2)$$

Note here that $P(\mu, \tau)$ is in fact the conditional probability $P(\mu, \tau | \mu_0)$ with μ_0 as the eigenvalue matrix of H_0 but for simplification we use the former notation. As the τ -dependence of P in eq.(2) enters only through λ_i , a derivative of P with respect to τ can be written as follows [10]

$$\frac{\partial P(\mu_i, \tau)}{\partial \tau} = - \sum_{n=1}^N \int \prod_{i=1 \neq n}^N \delta(\mu_i - \lambda_i) \frac{\partial \delta(\mu_n - \lambda_n)}{\partial \mu_n} \frac{\partial \lambda_n}{\partial \tau} \rho(V) dV \quad (3)$$

The further treatment of above equation depends on the symmetry-classes of the matrices V and H , that is, whether they are real-symmetric or complex hermitian.

In the real-symmetric case, $\frac{\partial \lambda_n}{\partial \tau}$ can be expressed in terms of the matrix elements of the orthogonal matrix O which diagonalizes H , $H = O^T \Lambda O$ with Λ as the eigenvalue matrix (Appendix A). Using this in eq.(3) leads to the following form with $g_{kl} = (1 + \delta_{kl})$,

$$\frac{\partial P(\mu_i, \tau)}{\partial \tau} = \Omega^2 \sum_n \frac{\partial}{\partial \mu_n} (\mu_n P) - 2h^{-1} \Omega \sum_n \frac{\partial}{\partial \mu_n} \int \prod_i \delta(\mu_i - \lambda_i) \sum_{k \leq l} O_{nk} O_{nl} \frac{V_{lk}}{g_{kl}} \rho(V) dV \quad (4)$$

an application of the partial integration on the second term in the right of above equation then gives

$$\frac{\partial P(\mu_i, \tau)}{\partial \tau} = \Omega^2 \sum_n \frac{\partial}{\partial \mu_n} (\mu_n P) - h^{-1} \Omega \sum_n \frac{\partial}{\partial \mu_n} \int \sum_{k \leq l} \frac{1}{\alpha_{kl} g_{kl}} \frac{\partial}{\partial V_{kl}} \left[\prod_i \delta(\mu_i - \lambda_i) O_{nk} O_{nl} \right] \rho(V) dV \quad (5)$$

For simplification let us assume a same variance for all the diagonal matrix elements such that $g_{kl} \alpha_{kl} = \alpha$ for $k = l$. Now by expressing $\sum_{k \leq l} \frac{1}{\alpha_{kl} g_{kl}} \dots = \frac{1}{\alpha} \sum_{k \leq l} \dots - \sum_{k < l} \left(\frac{1}{\alpha} - \frac{1}{g_{kl} \alpha_{kl}} \right)$ and with the help of real-symmetric analog of relations (A3-A8) as well as the partial integration technique we can reduce eq.(5) as follows

$$\begin{aligned} \frac{\partial P(\mu_i, \tau)}{\partial \tau} &= \Omega^2 \sum_n \frac{\partial}{\partial \mu_n} (\mu_n P) + \frac{\Omega}{h\alpha} \sum_n \frac{\partial}{\partial \mu_n} \sum_m \frac{\partial}{\partial \mu_m} \int \prod_i \delta(\mu_i - \lambda_i) \sum_{k \leq l} \left[\frac{\partial \lambda_m}{\partial V_{kl}} O_{nk} O_{nl} \right] \rho(V) dV \\ &\quad + \frac{1}{\alpha} \sum_{n, m, n \neq m} \frac{\partial}{\partial \mu_n} \int \prod_i \delta(\mu_i - \lambda_i) \frac{\rho(V)}{\lambda_m - \lambda_n} dV \\ &\quad - h^{-1} \Omega \sum_{k < l} (g_{kl}^{-1} \alpha_{kl}^{-1} - \alpha^{-1}) \sum_n \frac{\partial}{\partial \mu_n} \int \left[\frac{\partial}{\partial V_{kl}} \left(\prod_i \delta(\mu_i - \lambda_i) O_{nk} O_{nl} \right) \right] \rho(V) dV \end{aligned} \quad (6)$$

By applying the orthogonality relation of matrix O , the eq.(6) can now be rewritten as follows

$$\frac{\partial P(\mu_i, \tau)}{\partial \tau} = \Omega^2 \sum_n \frac{\partial}{\partial \mu_n} (\mu_n P) + \frac{1}{\alpha} \sum_n \frac{\partial}{\partial \mu_n} \left[\frac{\partial}{\partial \mu_n} + \sum_{m \neq n} \frac{\beta}{\mu_m - \mu_n} \right] P - F \quad (7)$$

where $\beta = 1$ and

$$F = h^{-2} \Omega^2 \sum_{k < l} (g_{kl}^{-1} \alpha_{kl}^{-1} - \alpha^{-1}) \int \prod_i \delta(\mu_i - \lambda_i) [\alpha_{kl} - 2\alpha_{kl}^2 V_{kl}^2] \rho(V) dV \quad (8)$$

Following the similar steps and the appropriate relations given in Appendix A, one obtains the same equation in complex hermitian case too (with $\beta = 2$). The further analysis of the eq.(7) depends on the relative values of variances for the off-diagonal matrix elements. For a clear understanding, let us first consider a case where all the off-diagonals have the same variances, such that $g_{kl} \alpha_{kl} = \alpha'$ (for $k \neq l$). F then can be expressed in terms of the derivative of P with respect to $y \equiv \frac{\alpha'}{\alpha}$,

$$F = 2h^{-2} \Omega^2 (1 - y) y \frac{\partial P}{\partial y} \quad (9)$$

Here we have used an idea quite important to our analysis that the variances of the matrix elements of V can also vary and therefore eigenvalues evolve not only as a function of the perturbation strength but the variances too. Note here that F can also be expressed as an extra drift term ($F = \sum_n \frac{\partial R(\mu)}{\partial \mu_n}$), with $R = \int \left[\sum_{k \leq l} (g_{kl}^{-1} \alpha_{kl}^{-1} - \alpha^{-1}) \frac{\partial}{\partial V_{kl}} \left(\prod_i \delta(\mu_i - \lambda_i) O_{nk} O_{nl} \right) \right] \rho(V) dV$ and it can be of interest to study the effect of this extra drift on the dynamics. For example, for R nonlinear in P , the motion may loose its random behaviour, showing some sort of periodicity. But in this paper, we restrict ourselves just to explore the universality of the dynamics, that is, to verify that P satisfies a similar F-P equation as the one proposed by Dyson.

The eq.(7) contains the derivatives of P with respect to two parameters, namely, τ and y . In order to prove that the statistical quantities in the GG case evolve in a similar way as in the SG case, one should be able to reduce the two

parametric dependence of eq.(7) in a single parameter. We assume that it is possible along a curve parameterized by ϕ , $\tau = \tau(\phi)$, $y = y(\phi)$ such that

$$\frac{\partial}{\partial \phi} = \frac{\partial \tau}{\partial \phi} \frac{\partial}{\partial \tau} + \frac{\partial y}{\partial \phi} \frac{\partial}{\partial y} \quad (10)$$

A comparison of eq.(10) with eq.(7) then leads us to the conditions for existence of such a curve namely, $\frac{\partial \tau}{\partial \phi} = 1$ and $\frac{\partial y}{\partial \phi} = \frac{2\Omega^2 y(1-y)}{h^2}$. By solving these equations, one obtains the following curve in the (y, τ) -space,

$$y = \frac{e^{2\Omega^2 \tau} - 1}{e^{2\Omega^2 \tau} + y_0} \quad \text{and} \quad \tau = \phi + \tau_0 \quad (11)$$

where τ_0 and y_0 are arbitrary constants. Note the conditions mentioned above do not impose any constraints on these constants except that the curve should remain on the positive side of the upper-half of $y - \tau$ plane. The extraction of τ_0 from eq.(11) leads to the following form of ϕ

$$\phi = \tau - \frac{1}{2\Omega^2} \log \left[1 + d \frac{|y-1|}{y} (e^{2\Omega^2 \tau} - 1) \right] \quad (12)$$

where $d = \frac{y_0}{y_0-1}$ and can be chosen as unity (as the corresponding value of ϕ still satisfies the required conditions). Thus, for $y = \frac{\alpha'}{\alpha}$, eq.(11) represents a set of curves along which eq.(7) adopts the form of a F-P equation

$$\frac{\partial P(\mu_i, \phi)}{\partial \phi} = \Omega^2 \sum_n \frac{\partial}{\partial \mu_n} (\mu_n P) + \frac{1}{\alpha} \sum_n \frac{\partial}{\partial \mu_n} \left[\frac{\partial}{\partial \mu_n} + \sum_{m \neq n} \frac{\beta}{\mu_m - \mu_n} \right] P \quad (13)$$

here the steady state is achieved for $\phi \rightarrow \infty$ which corresponds to $\tau \rightarrow \infty$ and $\alpha \rightarrow \alpha'$; the steady state solution is given by $\prod_{i < j} |\mu_i - \mu_j|^{\beta} e^{-\alpha \Omega^2 \sum_k \mu_k^2}$. Note that only $\tau \rightarrow \infty$ (with $\frac{\partial P}{\partial \tau} = 0$) no longer represents the steady state as in SG or WD case but represents a transition state with $\phi(y) = -\frac{1}{2\Omega^2} \log(d \frac{|y-1|}{y})$.

The steady state limit of P at $(y, \tau) = (1, \infty)$ seems to indicate that if the initial perturbation is such that the width ($\propto \frac{1}{\alpha}$) of the eigenfunctions of H is smaller than the overlapping between them ($\propto \frac{1}{\alpha'}$), they have a tendency to overcome this difference and extend over whole of the available Hilbert space. This also seems to suggest that a complicated interaction giving rise to larger transition probabilities between energy levels as compared to the probability of staying in the same level can only be transient in nature. Given freedom to change, it rearranges itself in such a way so as to equalize these probabilities.

This can be seen directly from eqs.(7,8) because in $N \rightarrow \infty, \Omega \rightarrow 0$ and $H \rightarrow V$ thus removing any Ω -dependence of λ_i 's and making $F \rightarrow 0$. A same result, for SG type perturbation in large N -limit, has already been obtained [9]. The analogous behaviour in both GG and SG case can be attributed to the negligible effect of the distribution of N -diagonal matrix elements on the dynamics in presence of $N(N-1)/2$ off-diagonal ones. This indicates that, in the thermodynamic limit, difference of the variances does not leave its trace on the evolution of eigenvalues.

Again, in $N \rightarrow \infty$ limit, all the three distributions, namely, SG, GG and WD evolve in the same way for arbitrary initial conditions. Thus all moments of the eigenvalues in SG and GG cases at any value of the transition parameter will be equal to the corresponding moments of particle positions of a WD gas undergoing Brownian motion due to the presence of thermal noise. However, as discussed in [9], this does not imply a Brownian motion of eigenvalues for the first two case. This is due to the difference of sources randomizing the motion of eigenvalues. For SG and GG cases, the source is the matrix V , acting like a quenched disorder while, for WD gas, the thermal noise gives rise to an annealed randomness. Although these different origins of randomness do not affect the static (for one parameter value) as well as the 2^{nd} -order parametric correlations but, as shown in ref.[9], the multiple parametric correlations higher than the 2^{nd} -order are different for the perturbed systems and WD gas. However, the origin of randomness being similar for the SG and GG cases, all the conclusions obtained for the correlations for the former [9] are also valid for the latter in the thermodynamic limit.

It is interesting to note that eq.(4) can also be written in the following form (by first using the real-symmetric analog of eq.(A3) in eq.(4) and then taking the derivative with respect to μ_n inside the integral)

$$\frac{\partial P(\mu_i, \tau)}{\partial \tau} = \Omega^2 \sum_n \frac{\partial}{\partial \mu_n} (\mu_n P) + h^{-2} \Omega^2 \sum_{k \leq l} \int \frac{\partial \prod_i \delta(\mu_i - \lambda_i)}{\partial V_{kl}} V_{kl} \rho(V) dV \quad (14)$$

Now an application of the partial integration to the second term on the right of this equation gives

$$\int \frac{\partial \prod_i \delta(\mu_i - \lambda_i)}{\partial V_{kl}} V_{kl} \rho(V) dV = \int \prod_i \delta(\mu_i - \lambda_i) [1 + 2\alpha_{kl} V_{kl}^2] \rho(V) dV \quad (15)$$

By expressing this in terms of derivatives of P with respect to α, α' as before, we obtain a drift equation (without a diffusion term as well as the repulsive pairwise potential)

$$\frac{\partial P(\mu_i, \tau)}{\partial \phi_1} = \Omega^2 \sum_n \frac{\partial}{\partial \mu_n} (\mu_n P) \quad (16)$$

where ϕ_1 describes a curve in a 3-dimensional (y, α, α') space such that $\frac{\partial P}{\partial \phi_1} = \frac{\partial P}{\partial \tau} + \frac{2\Omega^2}{h^2} [\alpha' \frac{\partial P}{\partial \alpha'} + \alpha \frac{\partial P}{\partial \alpha}]$. The above possibility of reduction of the F-P equation in a pure drift equation just by going to a higher parametric space implies the total blindness of eigenvalues to any mutually repulsive potential or locally fluctuating force in this space. Note that it is already well-known that a repulsion between the eigenvalues vanishes if the number of parameters of the hamiltonian undergoing variation is more than one in real-symmetric case (two for complex-hermitian case) [11]. However the same situation seems to exist even when the parameters undergoing variation are the ones governing the distribution of matrix elements of hamiltonians.

So far we have considered the case where all the off-diagonal matrix elements V_{ij} have same variance although different from that of diagonal. But in many physical situations e.g. in disordered systems, there occurs localization of eigenfunctions which may give rise to off-diagonal elements with different variances. One such situation most often encountered is where V_{ij} 's decay exponentially with respect to the distance from diagonal. It can be modeled by taking different variances for different off-diagonals, that is, $g_{kl}\alpha_{kl} = \alpha_r$, ($r = |k - l|$). F in such a case is given by, with $y_r \equiv \frac{\alpha_r}{\alpha}$

$$F = 2h^{-2}\Omega^2 \sum_{r=1}^{N-1} (1 - y_r) y_r \frac{\partial P}{\partial y_r} \quad (17)$$

We assume that it is possible along a curve parameterized by ϕ , $\tau = \tau(\phi)$, $Y = Y(y_1, \dots, y_N)$ such that

$$\frac{\partial}{\partial \phi} = \frac{\partial \tau}{\partial \phi} \frac{\partial}{\partial \tau} + \frac{\partial Y}{\partial \phi} \frac{\partial}{\partial Y} \quad (18)$$

with $\frac{\partial}{\partial Y} = \sum_r \frac{\partial y_r}{\partial Y} \frac{\partial}{\partial y_r}$. Reasoning again as before one finds that the universality of evolution of the probability density still survives but only by going to a higher (N -dimensional) parametric space (τ, y_1, \dots, y_N) , along a set of curves parametrized by ϕ and given by the conditions $\frac{\partial \tau}{\partial \phi} = 1$, $\frac{\partial Y}{\partial \phi} = 2h^{-2}\Omega^2$ with $\frac{\partial y_r}{\partial Y} = (1 - y_r)y_r$. Again ϕ can be obtained by solving these equations,

$$\phi = \tau - \frac{1}{2\Omega^2} \log \left[1 + d \prod_{r=1}^N \left(\frac{|y_r - 1|}{y_r} \right) (e^{2\Omega^2 \tau} - 1) \right] \quad (19)$$

Note here that \prod_r contains the contribution only from those y_r 's for which $y_r - 1 \neq 0$.

One can also consider a physical situation when all the off-diagonal matrix elements have different probability laws $g_{kl}\alpha_{kl} = \alpha$, (for $k = l$) and $\alpha_{kl} \neq \alpha_{ij}$ if $(kl \neq ij)$. In this case, F now turns out to be as follows with $y_{kl} \equiv \frac{\alpha_{kl}}{\alpha}$,

$$F = 2h^{-2}\Omega^2 \sum_{k < l} (1 - y_{kl}) y_{kl} \frac{\partial P}{\partial y_{kl}} \quad (20)$$

Now one has to go to $\frac{N(N-1)}{2} + 1$ -dimensional parametric space to recover the F-P equation similar to Wigner-Dyson gas. The curves in this case are again parameterized by ϕ which is still related to τ and Y by eq.(18) but now $Y \equiv Y(\{y_{kl}\})$ is such that $\frac{\partial}{\partial Y} = \sum_{k < l} \frac{\partial y_{kl}}{\partial Y} \frac{\partial}{\partial y_{kl}}$ with $\frac{\partial y_{kl}}{\partial Y} = (1 - y_{kl})y_{kl}$; the two other derivatives of τ and Y with respect to ϕ remain the same as in the above case. Proceeding exactly as before, ϕ in this case can be shown to be following,

$$\phi = \tau - \frac{1}{2\Omega^2} \log \left[1 + d \prod_{k < l} \left(\frac{|y_{kl} - 1|}{y_{kl}} \right) (e^{2\Omega^2 \tau} - 1) \right] \quad (21)$$

Again, as before, the $\prod_{k < l}$ contains contributions from all $y_{kl} \neq 1$.

For SG case, ϕ appearing in the correlators of the type $< d(E, \phi), d(E, 0) >$ involves variation of just one parameter, for CG case, two parameters or more. However, in both the cases, the expression of second order correlators is same due to same form of the F-P equation. Our study therefore suggests that the second order correlators involving variation of more than one parameter can always be expressed as those involving just one effective parameter. For example the correlators belonging to RM models of both localized as well as delocalized case can be expressed in the same form by using parameter ϕ although the definition of ϕ is different in two cases.

Our study also suggests that nature of the dynamics of the eigenvalues is very sensitive to the number of parameters undergoing a change. For finite N , the motion may appear as pure drift suggesting no interaction between eigenvalues and a total absence of local fluctuating and frictional forces if variances of all the matrix elements are allowed to vary independently. It seems Brownian as a function of just the relative variations of the variance. Under variation of only perturbation strength parameter, the presence of an extra drift like term makes the motion non-Brownian. But the contribution of this term being zero in $N \rightarrow \infty$ limit, it recovers its Brownian nature. It can be of interest to study the effect of this extra drift on the dynamics.

Although the study presented here deals with the energy level-dynamics of random matrix models of hermitian operators e.g. hamiltonian of complex systems, the results are also valid for the level-dynamics of unitary operators U e.g time-evolution operator being perturbed by a random matrix V taken from the GG ensembles, ($U(\tau) = U_0 \exp[i\sqrt{\tau}V] \approx U_0[1 + i\sqrt{\tau}V]$ for small τ values). The required F-P equation in this case turns out to be the following

$$\frac{\partial P(\mu_i, \phi)}{\partial \phi} = \frac{1}{\alpha} \sum_n \frac{\partial}{\partial \mu_n} \left[\frac{\partial}{\partial \mu_n} + \frac{\beta}{2} \sum_{m \neq n} \cot \left(\frac{\mu_m - \mu_n}{2} \right) \right] P \quad (22)$$

with ϕ defined as before. To prove the equivalence for U a symmetric unitary matrix ($\beta = 1$), one has to follow the similar steps as in the real-symmetric case of hermitian matrices as both are invariant under orthogonal transformation. Similarly for a general unitary U ($\beta = 2$), the steps are similar to the complex hermitian case. As obvious from eq.(22), this is similar to the F-P equation obtained if V belonged to the SG ensembles [12].

III. CALCULATION OF $P(\mu, \tau)$

In the preceeding section, we obtained a F-P equation governing the evolution of probability $P(\mu, \tau)$ for an arbitrarily given $H(\phi_0)$ (the conditional probability). Here ϕ_0 corresponds to the case $\tau = 0$ and $H(\phi_0)$ can be obtained by using the relation $H(\phi) = H(\phi_0) e^{-\Omega^2(\phi - \phi_0)} + \hat{V} \sqrt{1 - e^{-2\Omega^2(\phi - \phi_0)}}$, substituting ϕ in terms of τ and y there and then comparing it with the expression for the hamiltonian $H(\tau)$ given by eq.(1). Here \hat{V} is a matrix with an invariant probability distribution ($\rho(\hat{V}) \propto e^{-\alpha \text{Tr} \hat{V}^2}$) and $H(\phi)$ can be written in the above form as it gives the correct evolution equation for the probability distribution.

The formal similarity of the F-P equation with that of WD gas case as well as SG case makes it easier to obtain the $P(\mu, \tau)$ at least for $\beta = 2$ case as the solution for the latter case is already known. The $P(\mu, \phi)$ (and therefore $P(\mu, \tau)$) can also be calculated directly from hamiltonian $H(\phi)$ by using sum of the matrices technique (that is, by evaluating a two- matrix integral) as for the SG case [4] because now both the component matrices have invariant distribution unlike eq.(1). For completeness purpose, We give here few of the steps, used in solving SG case, for our case; for details refer to [12].

As can readily be checked, both eq.(13) and (22) can also be written as follows (with $\alpha = 1$ for simplification)

$$\frac{\partial P(\mu_i, \phi)}{\partial \phi} = \sum_n \frac{\partial}{\partial \mu_n} |Q_N|^\beta \frac{\partial}{\partial \mu_n} \frac{P}{|Q_N|^\beta} \quad (23)$$

where $|Q_N|^\beta = |\Delta(\mu)|^\beta e^{-\omega^2 \sum_k \mu_k^2}$ with $\Delta(\mu) = \prod_{i < j} (\mu_i - \mu_j)$ for the hermitian case and $= \prod_{j < k} \sin \left(\frac{\mu_j - \mu_k}{2} \right)$ for the unitary case. The transformation $\Psi = P/|Q_N|^{\beta/2}$ allows us to cast eq.(23) in the suggestive form

$$\frac{\partial \Psi}{\partial \phi} = -\hat{H} \Psi \quad (24)$$

where, for the hermitian case, the 'Hamiltonian' \hat{H} turns out to be the Calogero-Moser Hamiltonian

$$\hat{H} = \sum_i \frac{\partial^2}{\partial \mu_i^2} - \frac{1}{2} \sum_{i < j} \frac{\beta(\beta - 2)}{(\mu_i - \mu_j)^2} + \frac{\Omega^4}{4} \sum_i \mu_i^2 \quad (25)$$

Similarly, for the unitary case, \hat{H} is the Calogero-Sutherland hamiltonian

$$\hat{H} = - \sum_i \frac{\partial^2}{\partial \mu_i^2} + \frac{\beta(\beta-2)}{16} \sum_{i \neq j} \text{cosec}^2(\mu_i - \mu_j) - \frac{\beta^2}{48} N(N-1) \quad (26)$$

With the parabolic-confining potential (or periodic boundary conditions) and under the requirement (to take account of the singularity in H) that the solutions vanish as $|\mu_i - \mu_j|^{\beta/2}$ when μ_i and μ_j are close to each other, \hat{H} , both in eq.(25) and (26), has well-defined (completely symmetric) eigenstates ζ_k and eigenvalues λ_k . This allows us to express the "state" Ψ and therefore $P(\mu, \phi|H_0)$ as a sum over eigenvalues and eigenfunctions of \hat{H}

$$P(\mu, \phi|H(\phi_0)) = \left| \frac{Q_N(\mu)}{Q_N(\mu_0)} \right|^{\beta/2} \sum_{k \geq 0} \exp[-\lambda_k(\phi - \phi_0)] \zeta_k(\mu) \zeta_k^*(\mu_0) \quad (27)$$

where $\mu_0 \equiv (\mu_{01}, \mu_{02}, \dots, \mu_{0N})$ are the eigenvalues of $H(\phi_0)$. The joint probability distribution $P(\mu, \phi)$ can then be obtained by integrating over all initial conditions

$$P(\mu, \phi) = \int P(\mu, \phi|\mu_0, \phi_0) P(\mu_0, \phi_0) d\mu_0 \quad (28)$$

which further leads to correlations using standard techniques [12,13].

The eqs.(27-28) represent the formal solutions of eq.(13) (or eq.(22)). To proceed further, one needs to know the eigenvalues and eigenfunctions of \hat{H} so as to express P in a compact form but these are explicitly known only for $\beta = 2$ case [8]. This is because for $\beta = 2$ the interaction term in eqs.(25-26) drops out and then P can be obtained explicitly. For CM model (eq.(25)), it is given as follows [4],

$$P(\mu, \phi|H(\phi_0); \beta = 2) \propto \left| \frac{\Delta(\mu)}{\Delta(\mu_0)} \right| \det[f_m(\mu_i - \mu_{0j}; \phi - \phi_0)]_{i,j=1 \dots N} \quad (29)$$

with $f_m(x - y; t) = \exp \left[-\frac{x e^{\Omega^2 t} - y}{e^{2\Omega^2 t} - 1} \right]$, and, for CS model [12],

$$P(\mu, \phi|H(\phi_0); \beta = 2) = \frac{1}{N!} \left| \frac{\Delta(\mu)}{\Delta(\mu_0)} \right| \det[f_s(\mu_i - \mu_{0j}; \phi - \phi_0)]_{i,j=1 \dots N} \quad (30)$$

where $f_s(x) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \exp[-k^2 \phi + i k x]$.

As we already know $P(\mu, \phi)$ and various correlations for WD case (for which $\phi = \tau$) starting from various initial conditions [12,13], one can obtain these measures for GG case just by substituting ϕ by its appropriate relationship with τ and variances of the perturbation-matrix elements. For example, for the case $H = V\Omega^{-1}$ (which corresponds to $H(\tau \rightarrow \infty)$ in eq.(1)) with different variances for different diagonals of V (eq.(17)), $\phi = -(2\Omega^2)^{-1} \log[\prod_{r=1}^N \frac{|y_r - 1|}{y_r}]$ (with d chosen to be unity for the same reason as in eq.(12)). Also note that, in this case, $\frac{\partial P}{\partial \tau} = 0$. The initial condition $\phi = 0$ now corresponds to all $y_r \rightarrow \infty$, thus implying the existence of only the diagonal elements of V and a Poisson distribution for $P(\mu_0, \phi = 0)$ ($\propto e^{-\Omega^2 \alpha} \sum_i V_{ii}^2$ with $V_{ii} = \mu_{0i}$). As $\phi \rightarrow \infty$ when $y_r \rightarrow 1$ for all r , the equilibrium distribution is therefore given by the standard Gaussian ensembles. This case thus corresponds to the Poisson \rightarrow GE transition in the standard Gaussian ensembles [12,13] with ϕ now as a transition parameter (replacing τ , the perturbation strength) with intermediate ensembles representing the cases for any choice of y_r 's. The two-point correlations for this transition for V belonging to the SG ensemble (complex-hermitian) have already been obtained [13]. It should be noted that, as for the SG ensembles, ϕ must be rescaled to see the smooth transition [14]. .

IV. POLYNOMIAL CASE

In section II, we showed the equivalence of the dynamics of the Wigner-Dyson gas with the evolution of the eigenvalues of a hamiltonian under a perturbation V taken from a generalized Gaussian ensembles. However the claim about universality of dynamics and density correlators can only be made if the equivalence is proved, at least in thermodynamic limit, for a V with distribution of more general nature. In this section, We attempt to verify the claim by taking $\rho(V)$ as the exponential of the polynomial form of V , $\rho(V) = C \exp(-\sum_{k \leq l} Q(V_{kl}))$ with $Q(x) = \sum_{r=1}^M \gamma_{kl}(r) x^{2r}$ (a polynomial of x with degree $2M$), C as the normalization constant (referred as

polynomial case I later on) and variances for diagonal and off-diagonal matrix elements chosen to be arbitrary. Note the universality of the correlators for the case $H = V$, with V distributed as in this case, has already been studied in Ref.[14] (which corresponds to the steady state limit of the study given here)

To obtain a F-P equation, we now need following equality which can be proved by using integration by parts,

$$\int f[V] V_{kl} \rho(V) dV_{kl} = \frac{1}{2\gamma_{kl}(1)} \int \frac{\partial f[V]}{\partial V_{kl}} \rho(V) dV - \sum_{r=2}^k r \frac{\gamma_{kl}(r)}{\gamma_{kl}(1)} \int V_{kl}^{(2r-1)} f[V] \rho(V) dV \quad (31)$$

By using the real symmetric analog of eq.(A1) in eq.(3), followed by above equality, we get

$$\begin{aligned} \frac{\partial P}{\partial \tau} = & \Omega^2 \sum_n \frac{\partial}{\partial \mu_n} (\mu_n P) - h^{-1} \Omega \sum_n \frac{\partial}{\partial \mu_n} \int \sum_{k \leq l} \frac{1}{\gamma_{kl}(1) g_{kl}} \frac{\partial}{\partial V_{kl}} \left[\prod_i \delta(\mu_i - \lambda_i) O_{nk} O_{nl} \right] \rho(V) dV \\ & + 2h^{-1} \Omega \sum_{r=2}^M r \sum_n \frac{\partial}{\partial \mu_n} \int \prod_i \delta(\mu_i - \lambda_i) \left[\sum_{k \leq l} \frac{\gamma_{kl}(r)}{\gamma_{kl}(1)} \frac{V_{kl}^{2r-1}}{g_{kl}} O_{nk} O_{nl} \right] \rho(V) dV \end{aligned} \quad (32)$$

Now for simplification let us consider a case where $g_{kl}\gamma_{kl}(r) = \gamma(r)$ if $(k = l)$ and $g_{kl}\gamma_{kl}(r) = \gamma'(r)$ if $(k \neq l)$.

Using the tools given in appendix A and by an extensive use of the integration by parts while dealing with nasty integrals, the first term appearing in above equation can now be reduced in the same form as in GG case. This results in the following

$$\frac{\partial P(\{\mu_i\}, \tau)}{\partial \tau} = \Omega^2 \sum_n \frac{\partial}{\partial \mu_n} (\mu_n P) + \frac{1}{\gamma'(1)} \sum_n \frac{\partial}{\partial \mu_n} \left[\frac{\partial}{\partial \mu_n} + \sum_{m \neq n} \frac{\beta}{\lambda_m - \lambda_n} \right] P - Z \quad (33)$$

where $\beta = 1$ and

$$Z = \frac{\Omega^2}{h^2} \left(\frac{1}{\gamma(1)} - \frac{1}{\gamma'(1)} \right) F - \frac{\Omega^2}{h^2} \sum_{r=2}^k r \left[\frac{\gamma'(r)}{\gamma'(1)} G_{r1} + \frac{\gamma(r)}{\gamma(1)} G_{r2} \right] \quad (34)$$

with F , G_{r1} , G_{r2} given as follows

$$F = \frac{1}{2} \sum_n \frac{\partial}{\partial \mu_n} \int \left[\sum_k \frac{\partial}{\partial V_{kk}} \left(\prod_i \delta(\mu_i - \lambda_i) \frac{\partial \lambda_n}{\partial V_{kk}} g_{kk} \right) \right] \rho(V) dV \quad (35)$$

$$G_{r1} = \sum_n \frac{\partial}{\partial \mu_n} \int \prod_i \delta(\mu_i - \lambda_i) \left[\sum_{k < l} \frac{\partial \lambda_n}{\partial V_{kl}} V_{kl}^{2r-1} \right] \rho(V) dV \quad (36)$$

$$G_{r2} = \sum_n \frac{\partial}{\partial \mu_n} \int \prod_i \delta(\mu_i - \lambda_i) \left[\sum_k \frac{\partial \lambda_n}{\partial V_{kk}} V_{kk}^{2r-1} \right] \rho(V) dV \quad (37)$$

We apply partial integration to F and rewrite it as follows,

$$F = \gamma(1) \sum_k \int \prod_i \delta(\mu_i - \lambda_i) \left[1 - \gamma(1) V_{kk}^2 - \sum_{r=2}^M r \gamma(r) V_{kk}^{2r} \right] \rho(V) dV + \sum_{r=2}^k r \gamma(r) G_{r2} \quad (38)$$

similarly G_{r1} and G_{r2} can be rewritten as follows,

$$G_{r1} = \sum_{k < l} \int \prod_i \delta(\mu_i - \lambda_i) \left[(2r-1) V_{kl}^{2r-2} - 2\gamma'(1) V_{kl}^{2r} - 2 \sum_{s=2}^M s \gamma'(s) V_{kl}^{2r+2s-2} \right] \rho(V) dV \quad (39)$$

$$G_{r2} = \sum_k \int \prod_i \delta(\mu_i - \lambda_i) \left[(2r-1) V_{kk}^{2r-2} - \gamma(1) V_{kk}^{2r} - \sum_{s=2}^M s \gamma(s) V_{kk}^{2r+2s-2} \right] \rho(V) dV \quad (40)$$

For the complex Hermitian case also, one obtains an equation similar to eq.(33) with $\beta = 2$ and Z given by eq.(34). Note that all the terms appearing in the expressions of F, G_{r1}, G_{r2} are of the type $\int \prod_i (\mu_i - \lambda_i) V^j \rho(V) dV$, and, as done in section II for GG case, they can easily be rewritten in terms of the derivatives with respect to $\gamma(r)$'s if $j \leq 2M$. It is difficult to do so for terms with $j > 2M$, they probably could be expressed as higher order derivatives with respect to more than one variance parameters (this is the case at least for $N = 2$). However as physically significant RM models of complex systems generally correspond to limit $N \rightarrow \infty$, it would be sufficient, for our purpose, to study the behaviour of the terms in this limit. As obvious from eqs.(38-40), the N or Ω -dependence of F, G_{r1} and G_{r2} is due to presence of the λ_i 's which can be clearly seen by using the equality $\delta(\lambda^{-1}[\mu] - V) = \delta(\mu - \lambda[V]) \det |\frac{\partial \lambda}{\partial V}|$ and rewriting these integral in terms of the function $\lambda^{-1}(\mu)$. Let us consider the case when both $\gamma(r)$ and $\gamma(r')$, $r = 1 \rightarrow M$, are independent of N and all the matrix elements of V are distributed with zero mean; the latter choice implies the N -independence of V_{kl} also. Thus the N -dependence of F, G_{r1} and G_{r2} in this case is of the same order as appears in P and therefore it follows from eq.(34) that the contribution from Z to eq.(33) is negligible (as compared to the diffusion term and the drift term due to mutual repulsion) in that limit. Similarly for GG case, Z vanishes for $N \rightarrow \infty$; Z here can be obtained by substituting $\gamma(r) = 0$, $\gamma'(r) = 0$, for $r > 1$, in eq.(13). Further for the case where distribution, although basis independent, contains a non-Gaussian term, that is, for $\rho(V) \propto \exp[-\alpha \text{Tr}(V^2) - \sum_{r=2}^M \gamma(r) \text{Tr}(V^{2r})]$ (polynomial case II), the Z can be shown to be following,

$$Z = \frac{2\gamma\Omega^2}{h^2} \sum_{r=2}^k r\gamma(r) \int \prod_i \delta(\mu_i - \lambda_i) \left[\sum_{k,l} g_{k,l} \frac{\partial (V^{2r-1})_{kl}}{\partial V_{kl}} - 4\alpha \text{Tr}(V^{2r}) - 4 \sum_{s=2}^k \text{Tr}(V^{2r+2s-2}) \right] \rho(V) dV \rightarrow 0 \quad \text{for } N \rightarrow \infty \quad (41)$$

In thermodynamic limit ($N \rightarrow \infty$), the eq.(33) is same as the F-P equation governing the Brownian motion of particles in Wigner-Dyson gas (also the Sutherland model) with particle positions and time in the latter replaced by μ_i and τ in the former. This remains valid also for the case when $Q(x)$ is chosen to be an arbitrary function expandable in a Taylor series. Thus we find that, under a perturbation taken from an ensemble with a sufficiently general distribution, the distribution of eigenvalues of the quantum system evolves in the same way as the distribution of particle position in the Wigner-Dyson gas for arbitrary initial conditions. This also implies that all the moments of the eigenvalues calculated for a parameter value τ will be equal to the corresponding moments of particle positions of the latter. But the motion of the eigenvalues is not Brownian because the randomness in their motion comes from the matrix V which acts like quenched disorder [9].

This also implies the equivalence of second order parametric correlators in the two cases because they can be expressed as a sum over various moments of eigenvalues, weighted suitably and then averaged over equilibrium initial conditions. However, the F-P equation being Markovian in nature, its equivalence for the two cases can not lead us to a similar conclusion for the higher order correlators which involve moments at more than one parameter value and therefore multiple parameter averaging. As mentioned in [9], the n -point correlations for the Wigner-Dyson gas can be expressed in terms of the two point functions, yielding an n -matrix integral while for RM models of quantum chaos, the n -point function remains a two matrix integral.

It should be noted here that we have considered the case only for N -independent coefficients in the polynomial; a more careful analysis is required when these have different N -dependence. For example, if one or more coefficients are N -dependent, increasing with N increasing, the contribution to eq.(33) from terms in eqs.(38-40) is no longer negligible and therefore the evolution of P is no longer the same as in Gaussian case.

V. CONCLUSION

In this paper, we have analytically studied the response of energy levels of complex quantum systems to external perturbations modeled by generalized random matrix ensembles. Our results indicate the universality of two-point parametric density correlators as well as the static correlators of all orders thus agreeing with the numerically observed results for complex systems. One interesting feature revealed by our study is that for both the localized and the delocalized quantum dynamics of these systems, the second order parametric correlations can be shown to have the same form except that the definition of the effective parameter is different in the two cases. The method adopted here handicaps us from making any statement about universality of the higher order parametric correlations.

Further, for the reasons given in the sections (I,IV), the correspondence shown between Wigner-Dyson gas and the quantum hamiltonian $H = H_0 + xV$ with V having a sufficiently general nature also implies the equivalence between the space-time correlators of the Calogero-Sutherland system and the second order parametric correlations

of H . This further strengthens the idea contained in [1] and supported by studies in [9], namely, the 1D Sutherland model provides a model hamiltonian for the dynamics of eigenvalues of quantum chaotic systems [10]. The equivalence shown between the F-P equation governing the evolution, of eigenvalues in GG and SG cases also turns out to be quite helpful in studying the correlations for various difficult but physically significant situations (depending on the variances of perturbation matrix) in the former case, by using the technique given in section III. Note that our result is quite general as we have shown the equivalence for arbitrary choice of variances. As already mentioned in section II, the results obtained in sections II, III and IV are also valid for the level-dynamics of unitary operators U . Note the analogy between the statistical properties of non-equilibrium Circular ensembles which model the eigenvalue spectra of unitary operators and non-equilibrium standard Gaussian ensembles has already been proved [12].

Our study still leaves many important questions unanswered. For example, universality or its absence among higher order correlations of complex systems and their similarities or differences with Wigner-Dyson gas is not fully understood. Further the results here have been obtained for explicit averaging over a generalized random perturbing potential. It would be interesting to know whether similar conclusions can also be obtained for the complex systems where ensemble averaging is not valid (e.g. Billiards) and eigenvalue statistics should be computed as an average over the N eigenvalues. The intuition suggests a positive answer as the sufficiently complex systems are quite likely to be self-averaging. This implies that a single choice of matrices from a particular distribution are representative in the large N -limit of the entire distribution and therefore an average over eigenvalues should give the same result as an ensemble average.

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APPENDIX A:

For H a complex hermitian matrix, its eigenvalue equation is given by $\Lambda = U.H.U^+$ with Λ as eigenvalue matrix and U the eigenvector matrix which is unitary. Now for a complex hermitian perturbation $V_{ij} = V_{ij;1} + iV_{ij;2} = V_{ji}^*$ (with $V_{ii;2} = 0$), the rate of change of eigenvalues and eigenvectors, with respect to various components of matrix elements of V , can be described as follows:

$$\frac{\partial \lambda_n}{\partial \tau} = -\Omega^2 \lambda_n + \frac{\Omega}{h} \sum_{i \leq j} \frac{1}{g_{ij}} [V_{ij;1}(U_{ni}U_{nj}^* + U_{nj}U_{ni}^*) + iV_{ij;2}(U_{ni}U_{nj}^* - U_{nj}U_{ni}^*)] \quad (\text{A1})$$

where

$$\frac{\partial H}{\partial \tau} = -\Omega^2 H + \frac{V\Omega}{h} \quad (\text{A2})$$

Further

$$\frac{\partial \lambda_n}{\partial V_{kl;1}} = \frac{h(\tau)}{\Omega g_{kl}} [U_{nk}U_{nl}^* + U_{nl}U_{nk}^*] \quad (\text{A3})$$

$$\frac{\partial \lambda_n}{\partial V_{kl;2}} = \frac{h(\tau)}{\Omega g_{kl}} [U_{nk}U_{nl}^* - U_{nl}U_{nk}^*] \quad (k \neq l) \quad (\text{A4})$$

$$\frac{\partial \lambda_n}{\partial V_{kl;2}} = 0 \quad (\text{if } k = l) \quad (\text{A5})$$

and

$$\frac{\partial U_{np}}{\partial V_{kl;1}} = \frac{-h(\tau)}{\Omega g_{kl}} \sum_{m \neq n} \frac{1}{\lambda_m - \lambda_n} U_{mp}(U_{mk}^*U_{nl} + U_{ml}^*U_{nk}) \quad (\text{A6})$$

$$\frac{\partial U_{np}}{\partial V_{kl;2}} = \frac{-h(\tau)}{\Omega g_{kl}} \sum_{m \neq n} \frac{1}{\lambda_m - \lambda_n} U_{mp} (U_{mk}^* U_{nl} - U_{ml}^* U_{nk}) \quad (\text{A7})$$

and

$$\sum_{k \leq l} \left[\frac{\partial (U_{nk} U_{nl}^* + U_{nl} U_{nk}^*)}{\partial V_{kl;1}} + i \frac{\partial (U_{nk} U_{nl}^* - U_{nl} U_{nk}^*)}{\partial V_{kl;2}} \right] = - \frac{2h(\tau)}{\Omega g_{kl}} \sum_{m \neq n} \frac{1}{\lambda_n - \lambda_m} \quad (\text{A8})$$

For the real-symmetric case, the corresponding relations can be obtained by using $U_{ij} = U_{ij}^*$ (as eigenvector matrix is now real-orthogonal) in eqs.(A1-A8) and taking $V_{ij;2} = 0$ for all values of i, j (can also be found in Ref.[9]).

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